

$$2 \sin^{-1}(-1) = -\frac{\pi}{2}$$

$$3 \tan^{-1}(5) = 78.690068^\circ$$

$$\pi = 180^\circ$$

$$= 78.690068^\circ$$

$$= \frac{78.690068 \times \pi}{180} = 1.373$$

$$= 1.37$$

$$4. \tan^{-1}\left[\tan\left(-\frac{2\pi}{13}\right)\right] = -\frac{2\pi}{13}$$

$$7 \sin(\theta) = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}$$

$$= 180 - 30 = 150^\circ = \frac{5\pi}{6}$$

$$8 \quad 4 \sin \theta + 5 = 3$$

$$4 \sin \theta = -2$$

$$\sin \theta = -\frac{1}{2}, \quad \theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \frac{7\pi}{6} + 2\pi n, \quad \theta = \frac{11\pi}{6} + 2\pi n$$

$$\theta = \frac{7\pi}{6}, \quad \frac{11\pi}{6}$$

$$9. 2\sin^2\theta - 3\sin\theta + 1 = 0 \quad [0, 2\pi]$$

$$\text{Let } x = \sin\theta$$

$$~~2x^2 - 3\sin\theta~~$$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x-1) - (x-1) = 0$$

$$(2x-1)(x-1) = 0$$

$$2x-1 = 0 \quad \text{or} \quad x-1 = 0$$

$$2x = 1 \quad \text{or} \quad x = 1$$

$$x = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = 1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{or} \quad \theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$11. ~~\tan^2(\theta) + 1 = \sec^2\theta~~$$

$$12. \sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \quad A = \frac{\pi}{4}, B = \frac{\pi}{3}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\begin{aligned}
 13 \quad \sin\left(\frac{\pi}{2} - \theta\right) &= \sin\left(\frac{\pi}{2}\right) \cos \theta - \cos\left(\frac{\pi}{2}\right) \sin \theta \\
 &= \cos \theta - (0) \sin \theta \\
 &= \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \sin\left(\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}(1)\right) \quad & A = \sin^{-1}\left(\frac{1}{2}\right), B = \cos^{-1}(1) \\
 &= \sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) \cos\left(\cos^{-1}(1)\right) - \cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) \sin\left(\cos^{-1}(1)\right) \\
 &= \left(\frac{1}{2}\right)(1) - \left(\frac{\sqrt{3}}{2}\right)(0) \\
 &= \frac{1}{2}
 \end{aligned}$$

$$15 \quad 5 \cos(2\theta) = -5 \cos \theta \quad [0, 2\pi]$$

$$5 \cos(2\theta) + 5 \cos \theta = 0$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1$$

$$5(2 \cos^2 \theta - 1) + 5 \cos \theta = 0$$

$$10 \cos^2 \theta - 5 + 5 \cos \theta = 0$$

$$\text{Let } x = \cos \theta$$

$$10x^2 + 5x - 5 = 0$$

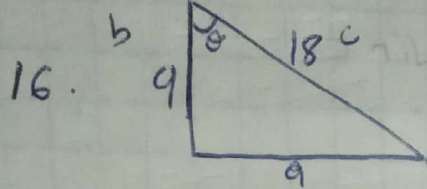
$$x_{1,2} = \frac{-5 \pm \sqrt{25 + 200}}{2(10)} = \frac{-5 \pm \sqrt{225}}{20} = \frac{-5 \pm 15}{20}$$

$$x = \frac{1}{2}, -1$$

$$\cos \theta = \frac{1}{2} \quad \text{or} \quad \cos \theta = -1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{or} \quad \theta = \pi$$

$$= \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$



$$18^2 = 9^2 + a^2$$

$$a^2 = 18^2 - 9^2 = 243$$

$$a = 9\sqrt{3}$$

$$\csc \theta = \frac{a}{\sqrt{3}}$$

$$\sin \theta = \frac{9\sqrt{3}}{18} = \frac{\sqrt{3}}{2}$$

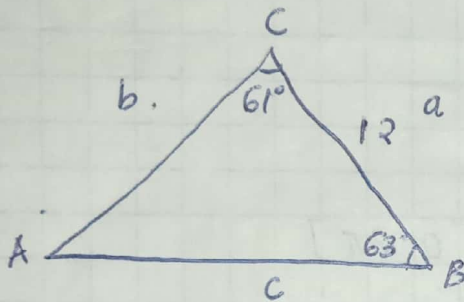
$$\cos \theta = \frac{9}{18} = \frac{1}{2}$$

$$\sec \theta = 2$$

$$\tan \theta = \frac{9\sqrt{3}}{9} = \sqrt{3}$$

$$\cot \theta = \frac{1}{\sqrt{3}}$$

17.



$$A = 180 - (61 + 63) = 56^\circ$$

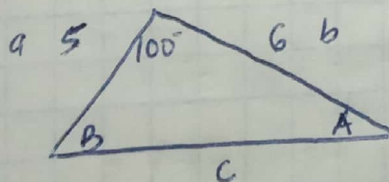
$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad ; \quad \frac{\sin 56}{12} = \frac{\sin 63}{b}$$

$$b = \frac{\sin 63}{\sin 56} \times 12 = 12.9$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \frac{\sin 56}{12} = \frac{\sin 61}{c}$$

$$c = \frac{\sin 61}{\sin 56} \times 12 = 12.66$$

18.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= 5^2 + 6^2 - 2(5)(6) \cos(100)$$

$$c = 8.45$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6^2 + 8.45^2 - 5^2}{2(6)(8.45)}$$

$$\cos A = 0.812648$$

$$A = 35.6^\circ$$

$$B = 180 - (35.6 + 100) = 44.4^\circ$$

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$$A = \cos^{-1} \left(\frac{b^2 + c^2 - a^2}{2bc} \right) = \cos^{-1} \left(\frac{18^2 + 20^2 - 17^2}{2(18)(20)} \right)$$

$$= 52.8^\circ$$

$$B = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2ac} \right) = \cos^{-1} \left(\frac{18^2 + 20^2 - 18^2}{2(17)(20)} \right)$$

$$= 57.5^\circ$$

$$C = 180 - (52.8 + 57.5) = 69.7^\circ$$

20.

$$p = \frac{8 + 17 + 15}{3} = \frac{40}{3} = 20$$

$$\text{Area} = \sqrt{p(p-a)(p-b)(p-c)}$$

$$= \sqrt{20(20-8)(20-17)(20-15)}$$

$$= 60$$

22 (a) $(-6, 4\pi)$

$$= (6, 4\pi - 5\pi) = (6, -\pi)$$

(b) $(-6, 4\pi)$

$$= (-6, 4\pi - 4\pi) = (-6, 0)$$

(c) $(-6, 4\pi)$

$$= (6, 4\pi - \pi) = (6, 3\pi)$$

$$23. \left(-1, -\frac{5\pi}{6}\right)$$

$$r = -1, \theta = -\frac{5\pi}{6}$$

$$= r(\cos \theta + i \sin \theta)$$

$$= -1 \left(\cos\left(-\frac{5\pi}{6}\right) + i \sin\left(-\frac{5\pi}{6}\right) \right)$$

$$= -1 \left(-\frac{\sqrt{3}}{2} + i \left(-\frac{1}{2}\right) \right)$$

$$= \frac{\sqrt{3}}{2} + i \frac{1}{2}$$

$$= \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$$

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$$r^2 = x^2 + y^2 = (-3\sqrt{3})^2 + 3^2$$

$$= 27 + 9 = 36$$

$$r = 6$$

$$\tan \theta = \frac{y}{x} = \left(\frac{3}{-3\sqrt{3}} \right) = \left(-\frac{1}{\sqrt{3}} \right)$$

$$\theta = \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) = -\frac{\pi}{6}$$

$$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$25. \quad r = \cos \theta$$

$$r(r) = r(\cos \theta)$$

$$r^2 = r \cos \theta$$

$$x^2 + y^2 = x$$

$$x^2 + y^2 - x = 0$$

$$r^2 = x^2 + y^2$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$26. \quad r \sin \theta = 6$$

$$y = r \sin \theta$$

$$y = 6$$

Horizontal line

$$27. \quad -5 + 9i$$

$$r^2 = x^2 + y^2 = 5^2 + 9^2 = 106$$

$$r = \sqrt{106}$$

$$\tan \theta = \frac{y}{x} = \frac{9}{-5}$$

$$\theta = \tan^{-1} \left(-\frac{9}{5} \right) = -60.95^\circ$$

$$180 - 60.95 = 119.05^\circ$$

$$= 119.1^\circ$$

$$29. \quad P = (9, 0), \quad Q = (0, 3)$$

$$\vec{PQ} = Q - P = (0, 3) - (9, 0) = (-9, 3)$$

$$= -9i + 3j$$

$$30. \quad L = \sqrt{1^2 + (-6)^2} = \sqrt{37}$$

$$\text{unit vector} = \frac{1}{\sqrt{37}} \langle 1, -6 \rangle$$

$$U = \frac{1}{\sqrt{37}} i - \frac{6}{\sqrt{37}} j$$

$$31. \quad \|V\| = 7, \quad \alpha = 45^\circ$$

$$V = 7 (\cos 45^\circ i + \sin 45^\circ j)$$

$$= \frac{7}{\sqrt{2}} i + \frac{7}{\sqrt{2}} j$$

$$32. \quad V = -3i - 8j$$

$$\tan \theta = \frac{-8}{-3}$$

$$\theta = \tan^{-1}\left(\frac{8}{3}\right) = 69.4^\circ$$

$$= 180 + 69.4 = 249.4^\circ$$

$$33. \quad V = 2j \quad W = i$$

$$\begin{aligned} (a) \quad V \cdot W &= (0i + 2j) \cdot (i - 0j) \\ &= (0 \cdot (1) + (2)(0)) = 0 \end{aligned}$$

$$(b) \quad \cos \theta = \frac{V \cdot W}{\|V\| \|W\|}$$

$$\|V\| = \sqrt{2^2} = 2 \quad \|W\| = \sqrt{1^2} = 1$$

$$\cos \theta = \frac{0}{2(1)} = 0$$

$$\theta = \cos^{-1}(0) = 90^\circ$$

(c) Orthogonal!

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$$V_1 = \frac{V \cdot W}{|W|^2} \cdot W$$

$$= \frac{(2i+j)(-6i-j)(-8i+j)}{(\sqrt{(-6)^2+(-1)^2})^2}$$

$$= \frac{(-12-1)}{37} (2i+j) = \frac{-13}{37} (-8i+j)$$

$$= 13$$

$$= \frac{78}{37} i + \frac{13}{37} j$$

$$\cancel{V_2 - V = V_1 = (-6i-j) - \left(\frac{78}{37}i + \frac{13}{37}j\right)}$$

$$V_2 = V - V_1 = (2i+j) - \left(\frac{78}{37}i + \frac{13}{37}j\right)$$

$$= -\frac{4}{37}i + \frac{24}{37}j$$

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$$(a) V \times W = \begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= i(1-3) - j(-3-0) + k(9-0)$$

$$= -2i + 3j + 9k$$

$$(b) W \times V = \begin{vmatrix} i & j & k \\ 0 & 3 & -1 \\ 3 & -1 & 1 \end{vmatrix}$$

$$= i(3-1) - j(0+3) + k(0-9)$$

$$= 2i - 3j - 9k$$

$$(c) \quad \mathbf{v} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 1 \\ 3 & -1 & 1 \end{vmatrix}$$

$$= \mathbf{i}(-1+1) - \mathbf{j}(3-3) + \mathbf{k}(-3+3)$$

$$= 0$$

$$(d) \quad \mathbf{w} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & -1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \mathbf{i}(-3+3) - \mathbf{j}(0-0) + \mathbf{k}(0-0)$$

$$= 0$$

$$36. \quad \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 3 \\ -4 & 2 & 2 \end{vmatrix}$$

$$= \mathbf{i}(-4-6) - \mathbf{j}(2+12) + \mathbf{k}(2-8)$$

$$= -10\mathbf{i} - 14\mathbf{j} - 6\mathbf{k}$$

$$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{(-10)^2 + (-14)^2 + (-6)^2}$$

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$$y = k - 9$$

$$= 2 - 4 = -2$$

$$y = -2$$

$$38. \quad \frac{x^2}{4} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

$$\frac{x^2}{2^2} + \frac{y^2}{5^2} = 1$$

$$\text{Vertices} = (0, 5), (0, -5)$$

$$= (0, 5), (0, -5)$$

39. $(-3, 2), (5, 2)$

foci = $(h-c, k), (h+c, k)$

centre = (h, k)

$$h-c = -3$$

$$h+c = 5$$

$$-2c = -8$$

$$c = 4$$

$$h-4 = -3$$

$$h = -3+4 = 1$$

$$h = 1$$

centre = $(1, 2)$

40. $\frac{(x+3)^2}{16} +$

$$40. \frac{(x+3)^2}{(7-3)^2} + \frac{(y-3)^2}{(6-3)^2} = 1$$

$$= \frac{(x+3)^2}{16} + \frac{(y-3)^2}{9} = 1$$

41. $\frac{(x-4)^2}{81} + \frac{(y+5)^2}{36} = 1$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 9$$

$$b = 6$$

$$(h, k) = (4, -5)$$

$$\text{Vertex (right)} = (h+a, k) = (9+4, -5) = (13, -5)$$

$$\text{Vertex (Left)} = (h-a, k) = (4-9, -5) = (-5, -5)$$

$$c^2 = a^2 + b^2 = 81 + 36 = 117$$

$$c = 3\sqrt{13}$$

$$\text{Focus (Right)} = (h+c, k) = (4+3\sqrt{13}, -5)$$

$$\text{Focus (Left)} = (h-c, k) = (4-3\sqrt{13}, -5)$$

$$44. \text{ Centre } = (h, k) = (2, 1) \quad a = 3$$

$$\begin{aligned} \text{Vertices} &= (h, k+a), (h, k-a) \\ &= (2, 1+3), (2, 1-3) \\ &= (2, 4), (2, -2) \end{aligned}$$

$$45. \text{ Centre } = (h, k) = (0, 0)$$

$$\text{Vertex} = (h+a, k) = (1, 0)$$

$$a = 1$$

$$\text{Focus} = (h+c, k) = (2, 0)$$

$$\frac{(x-0)^2}{1} - \frac{(y-0)^2}{3^2} = 1$$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

$$46 \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Vertices} = (-3, 0), (3, 0)$$

$$\text{Centre} = (0, 0)$$

$$a = 3$$

$$y = 2x \quad \frac{b}{a} = 2$$

$$\frac{b}{3} = 2$$

$$b = 6$$

$$\frac{x^2}{3^2} - \frac{y^2}{6^2} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

$$47 \quad \frac{y^2}{36} - \frac{x^2}{9} = 1$$

$$\frac{(y-0)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1$$

$$b = 6 \quad a = 3$$

$$a = 6, b = 3$$

$$\text{Vertices } (0, \pm a) = (0, \pm 6)$$

$$c = \sqrt{a^2 + b^2} = \sqrt{36 + 9} = 3\sqrt{5}$$

$$\text{foci} = (0, \pm c) = (0, \pm 3\sqrt{5})$$

$$y = \pm \frac{a}{b} x = \pm \frac{6}{3} x = \pm 2x$$

$$48. (y+2)^2 - 25(x+2)^2 = 25$$

$$\frac{(y+2)^2}{25} - (x+2)^2 = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$a=5$$

$$b=1$$

$$\text{Centre} = (h, k) = (-2, -2)$$

$$\text{Vertices} = (h, k-a), (h, k+a)$$

$$= (-2, -2-5), (-2, -2+5)$$

$$= (-2, -7), (-2, 3)$$

$$c = \sqrt{a^2 + b^2} = \sqrt{25+1} = \sqrt{26}$$

$$\text{foci} = (h, k+c), (h, k-c)$$

$$= (-2, -2+\sqrt{26}), (-2, -2-\sqrt{26})$$

Asymptotes

$$y = \pm \frac{a}{b} (x-h) + k$$

$$= \pm 5 (x+2) - 2$$

$$y = \pm 5(x+2) - 2$$

$$y+2 = \pm 5(x+2)$$